Efficient Frontier: Optimizing A Commodity Portfolio

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Abstract

In Modern Portfolio Theory (MPT), the "efficient frontier" represents the optimal asset allocation for a portfolio of multiple assets. The optimal portfolio is defined as one which maximizes its Sharpe ratio.

By optimizing a portfolio's asset allocation to have maximum Sharpe ratio, we maximize the amount of excess returns per unit of risk taken, thus creating an investment portfolio with the best risk-to-reward profile.

In this report, we apply this theory to a commodities-based portfolio, comprising of gold, silver, and crude oil. Historical monthly returns of these commodities, as well as US 3-month Treasury bills were taken, with a time frame of 35 years (Oct 1990 - Oct 2025).

The optimal asset allocation found was 9.4% silver, 78.6% gold, and 12.0% crude oil. The optimal portfolio outperformed all 3 individual assets in Sharpe ratio, whilst also delivering strong performance in a backtest.

Process

The analysis was conducted using Excel, with historical commodities price data sourced from macrotrends.com. Analysis tools used include Excel What-If-Analysis (10,000 simulations) and Excel Solver.

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Portfolio Construction:

Monthly returns data on silver, gold, crude oil, and US T-bills was imported into Excel. The first step in the analysis was calculating the monthly returns for each asset.

| | | Silver | | | | Gold | | | 0 | Crude Oil | | | ı | JS T-bill 3M | |
|-----------|-------|----------------|----------------|-----------|--------|----------------|----------------|-----------|-------|----------------|----------------|-----------|------------------|--------------------------|----------------|
| Date | Price | Monthly Return | Indexed Return | Date | Price | Monthly Return | Indexed Return | Date | Price | Monthly Return | Indexed Return | Date | Annualized Yield | Risk-free rate (monthly) | Indexed Return |
| 10/1/1990 | 4.22 | | 100.00 | 10/1/1990 | 379.25 | | 100.00 | 10/1/1990 | 35.31 | | 100.00 | 10/1/1990 | 7.240% | | 100.00 |
| 11/1/1990 | 4.09 | -3.081% | 96.92 | 11/1/1990 | 384.8 | 1.463% | 101.46 | 11/1/1990 | 29.08 | -17.644% | 82.36 | 11/1/1990 | 7.162% | 0.58% | 100.58 |
| 12/1/1990 | 4.19 | 2.445% | 99.29 | 12/1/1990 | 391 | 1.611% | 103.10 | 12/1/1990 | 28.48 | -2.063% | 80.66 | 12/1/1990 | 6.610% | 0.53% | 101.12 |
| 1/1/1991 | 3.83 | -8.592% | 90.76 | 1/1/1991 | 369.6 | -5.473% | 97.46 | 1/1/1991 | 21.9 | -23.104% | 62.02 | 1/1/1991 | 6.385% | 0.52% | 101.64 |
| 2/1/1991 | 3.68 | -3.916% | 87.20 | 2/1/1991 | 363 | -1.786% | 95.72 | 2/1/1991 | 19.28 | -11.963% | 54.60 | 2/1/1991 | 6.266% | 0.51% | 102.15 |
| 3/1/1991 | 3.84 | 4.348% | 91.00 | 3/1/1991 | 354 | -2.479% | 93.34 | 3/1/1991 | 19.63 | 1.815% | 55.59 | 3/1/1991 | 5.918% | 0.48% | 102.65 |
| 4/1/1991 | 3.93 | 2.344% | 93.13 | 4/1/1991 | 353.45 | -0.155% | 93.20 | 4/1/1991 | 20.99 | 6.928% | 59.44 | 4/1/1991 | 5.692% | 0.46% | 103.12 |
| 5/1/1991 | 4.13 | 5.089% | 97.87 | 5/1/1991 | 361.55 | 2.292% | 95.33 | 5/1/1991 | 21.16 | 0.810% | 59.93 | 5/1/1991 | 5.704% | 0.46% | 103.60 |
| 6/1/1991 | 4.45 | 7.748% | 105.45 | 6/1/1991 | 366.9 | 1.480% | 96.74 | 6/1/1991 | 20.56 | -2.836% | 58.23 | 6/1/1991 | 5.698% | 0.46% | 104.08 |
| 7/1/1991 | 4.07 | -8.539% | 96.45 | 7/1/1991 | 363.6 | -0.899% | 95.87 | 7/1/1991 | 21.7 | 5.545% | 61.46 | 7/1/1991 | 5.691% | 0.46% | 104.56 |
| 8/1/1991 | 3.82 | -6.143% | 90.52 | 8/1/1991 | 347 | -4.565% | 91.50 | 8/1/1991 | 22.28 | 2.673% | 63.10 | 8/1/1991 | 5.490% | 0.45% | 105.03 |
| 9/1/1991 | 4.14 | 8.377% | 98.10 | 9/1/1991 | 350.5 | 1.009% | 92.42 | 9/1/1991 | 22.25 | -0.135% | 63.01 | 9/1/1991 | 5.280% | 0.43% | 105.48 |
| 10/1/1991 | 4.09 | -1.208% | 96.92 | 10/1/1991 | 357.4 | 1.969% | 94.24 | 10/1/1991 | 23.29 | 4.674% | 65.96 | 10/1/1991 | 4.960% | 0.40% | 105.90 |
| 11/1/1991 | 4.07 | -0.489% | 96.45 | 11/1/1991 | 366.85 | 2.644% | 96.73 | 11/1/1991 | 21.48 | -7.772% | 60.83 | 11/1/1991 | 4.480% | 0.37% | 106.29 |
| 12/1/1991 | 3.86 | -5.160% | 91.47 | 12/1/1991 | 353.4 | -3.666% | 93.18 | 12/1/1991 | 19.15 | -10.847% | 54.23 | 12/1/1991 | 3.963% | 0.32% | 106.64 |
| 1/1/1992 | 4.13 | 6.995% | 97.87 | 1/1/1992 | 354.15 | 0.212% | 93.38 | 1/1/1992 | 18.93 | -1.149% | 53.61 | 1/1/1992 | 3.941% | 0.32% | 106.98 |
| 2/1/1992 | 4.12 | -0.242% | 97.63 | 2/1/1992 | 353.05 | -0.311% | 93.09 | 2/1/1992 | 18.69 | -1.268% | 52.93 | 2/1/1992 | 4.023% | 0.33% | 107.33 |
| 3/1/1992 | 4.12 | 0.000% | 97.63 | 3/1/1992 | 341.5 | -3.271% | 90.05 | 3/1/1992 | 19.49 | 4.280% | 55.20 | 3/1/1992 | 4.144% | 0.34% | 107.70 |
| 4/1/1992 | 3.98 | -3.398% | 94.31 | 4/1/1992 | 336.25 | -1.537% | 88.66 | 4/1/1992 | 20.88 | 7.132% | 59.13 | 4/1/1992 | 3.771% | 0.31% | 108.03 |
| 5/1/1992 | 4.06 | 2.010% | 96.21 | 5/1/1992 | 337.55 | 0.387% | 89.00 | 5/1/1992 | 22.13 | 5.987% | 62.67 | 5/1/1992 | 3.785% | 0.31% | 108.36 |
| 6/1/1992 | 4.03 | -0.739% | 95.50 | 6/1/1992 | 343.4 | 1.733% | 90.55 | 6/1/1992 | 21.84 | -1.310% | 61.85 | 6/1/1992 | 3.653% | 0.30% | 108.69 |
| 7/1/1992 | 3.95 | -1.985% | 93.60 | 7/1/1992 | 357.95 | 4.237% | 94.38 | 7/1/1992 | 21.83 | -0.046% | 61.82 | 7/1/1992 | 3.213% | 0.26% | 108.97 |
| 8/1/1992 | 3.72 | -5.823% | 88.15 | 8/1/1992 | 340.5 | -4.875% | 89.78 | 8/1/1992 | 21.46 | -1.695% | 60.78 | 8/1/1992 | 3.165% | 0.26% | 109.26 |
| 9/1/1992 | 3.77 | 1.344% | 89.34 | 9/1/1992 | 349 | 2.496% | 92.02 | 9/1/1992 | 21.83 | 1.724% | 61.82 | 9/1/1992 | 2.705% | 0.22% | 109.50 |
| 10/1/1992 | 3.76 | -0.265% | 89.10 | 10/1/1992 | 338.8 | -2.923% | 89.33 | 10/1/1992 | 20.68 | -5.268% | 58.57 | 10/1/1992 | 3.018% | 0.25% | 109.77 |
| 11/1/1992 | 3.73 | -0.798% | 88.39 | 11/1/1992 | 333.7 | -1.505% | 87.99 | 11/1/1992 | 19.91 | -3.723% | 56.39 | 11/1/1992 | 3.375% | 0.28% | 110.08 |
| 12/1/1992 | 3.67 | -1.609% | 86.97 | 12/1/1992 | 332.9 | -0.240% | 87.78 | 12/1/1992 | 19.49 | -2.109% | 55.20 | 12/1/1992 | 3.136% | 0.26% | 110.36 |
| 1/1/1993 | 3.71 | 1.090% | 87.91 | 1/1/1993 | 330.75 | -0.646% | 87.21 | 1/1/1993 | 20.27 | 4.002% | 57.41 | 1/1/1993 | 2.966% | 0.24% | 110.63 |
| 2/1/1993 | 3.57 | -3.774% | 84.60 | 2/1/1993 | 328.75 | -0.605% | 86.68 | 2/1/1993 | 20.53 | 1.283% | 58.14 | 2/1/1993 | 2.997% | 0.25% | 110.90 |
| 3/1/1993 | 3.89 | 8.964% | 92.18 | 3/1/1993 | 336.9 | 2.479% | 88.83 | 3/1/1993 | 20.44 | -0.438% | 57.89 | 3/1/1993 | 2.952% | 0.24% | 111.17 |
| 4/1/1993 | 4.3 | 10.540% | 101.90 | 4/1/1993 | 353.45 | 4.912% | 93.20 | 4/1/1993 | 20.54 | 0.489% | 58.17 | 4/1/1993 | 2.951% | 0.24% | 111.44 |
| 5/1/1993 | 4.71 | 9.535% | 111.61 | 5/1/1993 | 381 | 7.795% | 100.46 | 5/1/1993 | 20.04 | -2.434% | 56.75 | 5/1/1993 | 3.125% | 0.26% | 111.73 |
| 6/1/1993 | 4.58 | -2.760% | 108.53 | 6/1/1993 | 379 | -0.525% | 99.93 | 6/1/1993 | 18.82 | -6.088% | 53.30 | 6/1/1993 | 3.080% | 0.25% | 112.01 |
| 7/1/1993 | 5.3 | 15.721% | 125.59 | 7/1/1993 | 403.7 | 6.517% | 106.45 | 7/1/1993 | 17.94 | -4.676% | 50.81 | 7/1/1993 | 3.095% | 0.25% | 112.29 |

Fig 1. Monthly returns of silver, gold, crude oil, and US T-bills

We then calculate the mean and standard deviation of returns for each commodity, serving as our building block for the blended portfolio.

Additionally, a correlation matrix is constructed for each commodity relative to one another. This is critical for portfolio formation, as the fact that their correlation coefficients are less than 1 implies that there will be a diversification benefit.

| | Arithmetic Mean Return | Standard Deviation | Sharpe ratio |
|-----------|------------------------|--------------------|--------------|
| Silver | 0.914% | 8.109% | 0.086 |
| Gold | 0.657% | 4.414% | 0.100 |
| Crude Oil | 0.647% | 10.167% | 0.042 |
| T Bill | 0.216% | | |

Fig 2. Historical returns and standard deviation for commodities

| Correlations | Silver | Gold | Crude Oil |
|--------------|--------|-------|-----------|
| Silver | | | |
| Gold | 72.0% | | |
| Crude Oil | 24.8% | 10.1% | |

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Fig 3. Correlation matrix for commodities

Portfolio variance = $W_s^2 \sigma_s^2 + W_g^2 \sigma_g^2 + W_c^2 \sigma_c^2 + 2 W_s W_g \sigma_s \sigma_g \rho_{gg} + 2 W_s W_c \sigma_s \sigma_c \rho_{gc} + 2 W_g W_c \sigma_g \sigma_c \rho_{gc}$ Portfolio return = $W_s R_s + W_g R_g + W_c R_c$

Where:

 $W = weight, \, \sigma = standard \,\, deviation, \, \rho = correlation \,\, coefficient, \\ s = silver, \, g = gold, \, c = crude \,\, oil$

Efficient Frontier Creation:

We start by first assigning random weights to each asset in the portfolio, with the only constraint being that the weight of silver, gold, and crude oil must add up to 100%.

To create the efficient frontier graph, we utilize Excel's What-If Analysis tool to simulate 10,000 different possible portfolio allocations.

Each iteration of the simulation provides a different weightage of silver, gold, and crude oil, and consequently, a different portfolio return and portfolio standard deviation.

| Simulation | | |
|------------|-------------------|-----------------------|
| | Portfolio Std Dev | Portfolio Simulations |
| Simulation | 6.14% | 0.754% |
| 1 | 6.641% | 0.831% |
| 2 | 6.315% | 0.747% |
| 3 | 6.493% | 0.826% |
| 4 | 6.901% | 0.68896 |
| 5 | 6.762% | 0.729% |
| 6 | 5.734% | 0.719% |
| 7 | 5.204% | 0.718% |
| 8 | 4.41196 | 0.672% |
| 9 | 8.284% | 0.696% |
| 10 | 6.771% | 0.842% |
| 11 | 6.835% | 0.694% |
| 12 | 5.993% | 0.721% |
| 13 | 5.305% | 0.731% |
| 14 | 6.715% | 0.769% |
| 15 | 7.015% | 0.719% |
| 16 | 5.231% | 0.733% |
| 17 | 5.467% | 0.734% |
| 18 | 6.006% | 0.77296 |
| 19 | 6.094% | 0.753% |
| 20 | 4.617% | 0.675% |
| 21 | 5.911% | 0.790% |
| 22 | 5.441% | 0.746% |
| 23 | 4.424% | 0.671% |

Fig 4 Results for each iteration of the simulation

Using a scatter plot, the risk-reward profile of each allocation appears as such. Each dot on the plot below is one of the 10,000 simulated portfolio allocations.

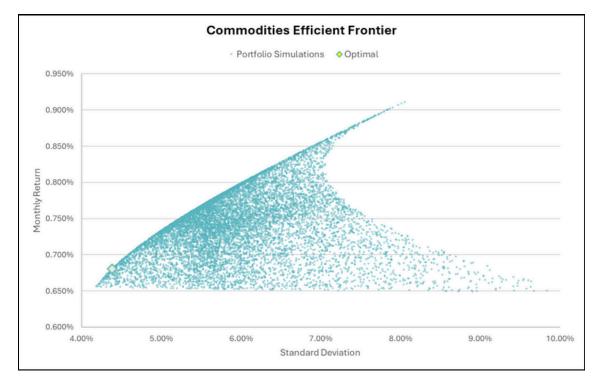


Fig 5. Scatter plot of all 10,000 simulated portfolio allocations

The higher up a point is on on the plot, the higher that allocation's expected return, while the further left they are on the plot, the lower its risk. Thus, The allocations with the best risk-to-reward profile are located towards the upper-left of the plot

Portfolio Optimization

The optimal portfolio is the portfolio with weightages of silver, gold, and crude oil such that the portfolio's Sharpe ratio (excess risk-adjusted return) is maximized.

Using Excel's Solver tool, we seek to maximize the portfolio's Sharpe ratio by adjusting the weights of each commodity in the portfolio.

We deduce the optimal commodities portfolio to be:

| 9.4% | 78.6% | 12.0% |
|--------|-------|-----------|
| Silver | Gold | Crude Oil |

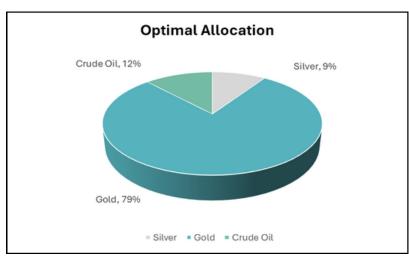


Fig 6. Optimal allocation of silver, gold, and crude oil.

The optimal commodity portfolio has a Sharpe ratio of 0.1059 (monthly), greater than all of the three individual commodities.

Due to the high allocation to gold (78.6%), the Sharpe ratio of the optimal portfolio ends up being only marginally higher than the Sharpe ratio of gold.

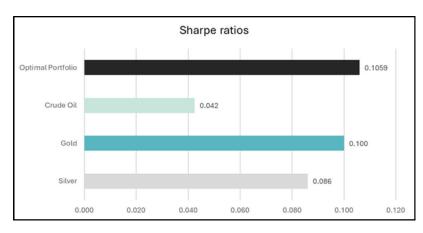


Fig 7. Comparison of Sharpe ratios, optimal portfolio vs individual commodities

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Backtest

A backtest of the optimal commodity portfolio was conducted, over the 35-year historical time frame (Oct 1990 - Oct 2025). The following results were obtained.

| | Arithmetic Mean Return | Geometric Mean Return | Standard Deviation | Max Monthly Drawdown | Max Monthly Gain |
|------------------------------|------------------------|-----------------------|--------------------|----------------------|------------------|
| Silver | 0.914% | 0.585% | 8.109% | -28.4% | 30.1% |
| Gold | 0.657% | 0.560% | 4.414% | -18.8% | 19.3% |
| Crude Oil | 0.647% | 0.136% | 10.167% | -54.2% | 85.0% |
| T Bill (Risk-free Rate) | 0.216% | 0.215% | | | |
| Optimal Portfolio (Backtest) | 0.680% | 0.583% | 4.385% | -21.3% | 6.7% |

Fig 8. Backtest of optimal portfolio using historial returns

Though the optimal portfolio did not have the highest returns, it did have the lowest volatility (standard deviation), and thus the highest risk-adjusted return (evidenced by higher Sharpe ratio).

Backtest Returns

By indexing the price of each commodity as well as the optimal portfolio to a base value of 100 in the year 1990, we can calculate the relative performance of each investment.

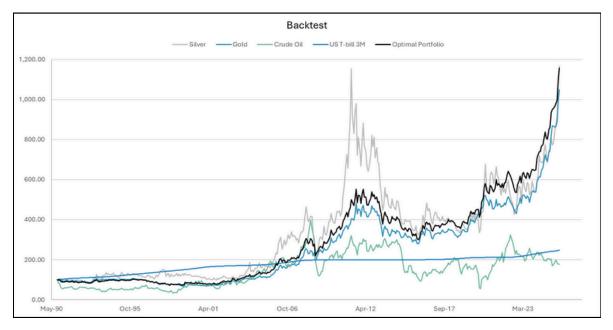


Fig 9. Indexed return of the various investments, October 1990 to October 2025

Silver had the highest absolute return, with the optimized portfolio coming in a close second, followed by gold in third, and oil in last place. However, the optimized portfolio, with its substantially lower standard deviation than silver, offered the best returns per unit of risk.

In terms of distribution of monthly returns, the optimized portfolio had a far narrower spread in its interquartile range of returns, as well as outlier values, as compared to the individual assets.

This highlights the diversification benefit of the optimized portfolio - on months where any of the individual assets has an extreme outlier return, the overall portfolio's return is likely to be less extreme due to the buffering effect of the other commodities in the portfolio.

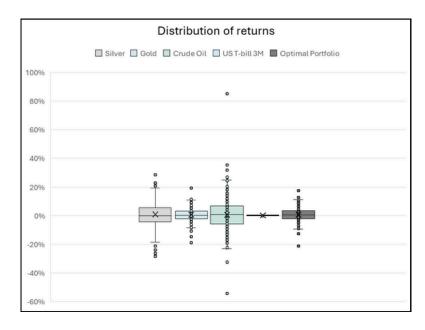


Fig 10. Median ECY from 1881 to 2025.

Takeaways

One notable observation for the optimal portfolio composition (9.4% silver, 78.6% gold, and 12.0% crude oil) is that despite silver having the highest CAGR in the period and crude oil having the lowest, crude oil still was allocated a higher portfolio weightage in the optimized portfolio, which may seem surprising. The underlying reason for this is due to crude oil's low correlation with the 2 other metals, thus providing substantial diversification benefits.

Overall, this test highlights how diversification can tangibly improve the risk-to-reward profile of an investors' portfolio, allowing them to capture upside growth whilst limiting downside losses.

References

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